Understanding Circular Motion

***Note this lesson was derived from and reproduced in part from the copyrighted Circular Motion Lab available on the internet at http://www.meador.org/inquiry.htm

Annotation
This inquiry lab is designed to help students understand the relationship between the variables in the equation for circular motion. Students will conduct three experiments. One variable in the circular motion equation is changed in each experiment. The conclusion of the lab involves deriving the circular motion equation using measured data. This lab is designed for physics students.

Primary Learning Outcome:
Circular Motion

Assessed QCC:
2.2 Investigates experimentally and solves problems that relate to time, distance, displacement, speed, velocity, and acceleration.
2.3 Resolves problems that involve motion vectors for direction and size.
3.3 Investigates experimentally and solves problems that relate gravitational forces, mass, distance, the Universal Gravitation constant and acceleration due to gravity.
3.4 Makes and analyzes graphs showing direct inverse, exponential relationships, and other variables.
5.1 Demonstrates the relationship among and solves problems that involve time, angular displacement, torque, rotational inertia, angular velocity, and angular acceleration for bodies in circular and rotary motion.

Non-Assessed QCC:
Deriving equations from measured data

Total Duration:
2.5 hours

Materials and Equipment:
6 inch long, 1 inch diameter PVC tube
Physics string (or and non stretching string)
Tape
Calculator
Stopwatch
Masses
5 rubber stoppers of the same size, material, and with the same number of holes
Masses that will add up to 100, 120, 150, 170, and 200 grams

Web Links:
http://www.meador.org/inquiry.htm URL:
Assessment:
The lab includes questions to be answered by the students and an answer key.

Remediation:
The lab ends by synthesizing three equations into one. This is the most difficult part of the lab and can be done as a teacher led classroom exercise.
The Circular Motion Lab

Answer questions in complete sentences

Introduction
We have discussed motion in straight lines and parabolic arcs. But many things move in circles or near circles, like the planets orbiting the sun and clothes in a dryer. To understand this type of motion, we must return to Newton’s First Law of Motion, the Law of Inertia.

1. State Newton’s first law of motion.

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________

2. The diagram on the right shows an overhead view of an object moving clockwise in a circular motion. The object is released at point P. Draw the subsequent motion of the body.

3. What are the two things which must be constant for an object to have a constant velocity?

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________
**Exploration**

Equipment for this lab includes a small tube, string, an assortment of masses, and a rubber stopper. Tie the rubber stopper to one end of the string. Thread the other end of the string through the tube. Tie the free end of the string to the 100 gram mass. Your gadget should look like the diagram shown below:

![Diagram of an apparatus with a rubber stopper on a string, tied to a 100 gram mass.]

Hold the tube with your left hand and grasp the 100 gram mass in your right hand. Gently twirl the tube so that the stopper begins spin in circular motion above your head. The diagram on the right shows a picture of the spinning stopper. Make the motion of the spinning stopper as perfectly horizontal (parallel to the ground) as you can. Now, adjust the speed of the stopper so that you can let go of the mass and it won't move up or down. Practice until you feel comfortable.

4. What is the force pulling the 100 gram mass down? This is the same force that is pulling the stopper inward. Show your equation, answer, and units below (Remember the unit for Newton is kg·m/s²).

_________________________________________________________________________

5. If the 100 gram mass is not moving up or down, what is the magnitude of the net force acting on it? (What is the magnitude of the vector sum of the forces?)

_________________________________________________________________________

6. You know that there is a force acting downward on the mass due to gravity. Draw a vector showing how this force acts on the stopper while in circular motion.

7. Draw the force that must exist to counterbalance the force of gravity and keep the stopper moving in a circular path with a constant radius. *Hint: think back to Newton’s first law.*
You will now perform three experiments to determine the relationship between the variables involved with circular motion.

**Experiment One: Speed (v) and Inward Acting Force (F_i)**

In this experiment you will keep the spinning radius constant and change the weight of the hanging mass.

A. Find the mass of the rubber stopper and record in Table 1.

B. Adjust the string so that the distance between the top of the tube and the middle of the stopper is 0.75m. Fasten a piece of masking tape to the string just below the bottom of the tube (but not touching). When you are whirling the apparatus, keep the tape just below the bottom of the tube to maintain a constant radius of 0.75m.

C. Again, twirl the tube so that the stopper travels in a horizontal circle. Adjust the speed of the stopper so that the tape stays just below the bottom of the tube. Make sure that the tape is not caught on the tube. Practice this until you feel comfortable keeping constant radius.

D. Now, you will measure the amount of time required for the stopper to make 10 revolutions with a radius of 0.75m. One member of the group will rotate the stopper. Another member will measure the time (using a stopwatch) and count the number of revolutions. Practice before you collect any data.

E. Measure the time to complete 10 revolutions twice.

F. Repeat with 120, 150, 170, and 200 g masses. Record your data in Table 1. Show your work for calculating the circumference.

<table>
<thead>
<tr>
<th>Table 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constants:</strong> 0.75m radius</td>
</tr>
<tr>
<td>Circumference of stopper _______m</td>
</tr>
<tr>
<td>Stopper mass _______kg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hanging mass (kg)</th>
<th>Weight of mass (N)</th>
<th>Time for 10 revolutions (s)</th>
<th>Average time for 1 revolution</th>
<th>Speed of stopper (ms⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trial 1</td>
<td>Trial 2</td>
<td>Average</td>
</tr>
<tr>
<td>0.100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.170</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use a calculator or computer to graph the inward force (the force or gravity) against the speed of the stopper. Put the speed on the horizontal axis and the force on the vertical axis. Select a power fit trend line for your data and display the equation on the graph. Round the coefficient to the nearest whole number.

8. Write the equation of your line \( [Y = \text{inward force} (F_i) \text{ and } X = \text{speed} (v)] \)

________________________________________________________________________

9. What does this equation tell you about the relationship between speed and the inward acting force?

________________________________________________________________________

________________________________________________________________________

**Experiment Two: Speed \( (v) \) and Radius \( (r) \)**

In this experiment you will find the speed of the circling stopper for different radii. All other variables will be held constant.

A. Begin by marking a radius of 1.50m on the string. Twirl the stopper and record the time that was required to make 10 revolutions while keeping the radius constant.

B. Repeat with radii of 1.25, 1.00, 0.75, 0.50m. Complete Table 2. If the times for a given radius are not close, take a third reading.

Table 2.
Constants: 0.100kg hanging mass
Stopper mass ______kg

<table>
<thead>
<tr>
<th>Radius (m)</th>
<th>Time for 10 revolutions (s)</th>
<th>Average time for 1 revolution</th>
<th>Circumference</th>
<th>Speed of stopper ( (\text{ms}^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trial 1</td>
<td>Trial 2</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use a calculator or computer to graph the radius against the speed of the stopper. Put the speed on the horizontal axis and the radius on the vertical axis. Select a power fit trend line for your data and display the equation on the graph. Round the coefficient to the nearest whole number.

10. Write the equation of your line \([Y = \text{radius} \ (r) \text{ and } X = \text{speed} \ (v)]\)

11. What does this equation tell you about the relationship between speed and radius?

_________________________________________________________________________

_________________________________________________________________________

Experiment Three: Speed \(v\) and the Circling Mass \(m\)

Now you will find the speed of the circling stopper for various weights of circling masses. All other variables will remain constant.

A. Attach one stopper to the end of the string and a 400g mass to the other. Set the radius at 0.5m. Twirl the stopper and record the time required to make 10 revolutions.

B. Repeat with 2, 3, 4, and 5 stoppers. Each of the stoppers should be the same size, material, and have the same number of holes. Complete Table 3. If the times for a given number of stoppers are not close, take a third reading.

Table 3.

<table>
<thead>
<tr>
<th>Constants:</th>
<th>0.50m Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inward force ___N</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of stoppers</th>
<th>Circling Mass</th>
<th>Time for 10 revolutions (s)</th>
<th>Average time for 1 revolution</th>
<th>Speed of stoppers (ms(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trial 1</td>
<td>Trial 2</td>
<td>Average</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use a calculator or computer to graph the radius against the speed of the stopper. Put the speed on the horizontal axis and the circling mass on the vertical axis. Select a power fit trend line for your data and display the equation on the graph. Round the coefficient to the nearest whole number.

12. Write the equation of your line \[ Y = \text{circling mass}(m) \text{ and } X = \text{speed}(v) \]

13. What does this equation tell you about the relationship between speed and radius?

Putting it all together

14. You will now try to combine your three equations into one that relates \( r, m, \) and \( v, \) to \( F. \) Your result should show \( F \) by itself on one side of the equation. Discuss this with your group.

15. Explain why an object moving in a circular path has a constant speed but is accelerating at the same time.
Understanding Circular Motion (Teacher's Copy)

Answer questions in complete sentences

**Introduction**

We have discussed motion in straight lines and parabolic arcs. But many things move in circles or near circles, like the planets orbiting the sun and clothes in a dryer. To understand this type of motion, we must return to Newton’s First Law of Motion, the Law of Inertia.

1. State Newton’s first law of motion.

An object at rest tends to stay at rest and an object in motion tends to stay in motion, with the same speed and in the same direction, unless acted upon by an unbalanced force.

2. The diagram on the right shows an overhead view of an object moving clockwise in a circular motion. The object is released at point P. Draw the subsequent motion of the body.

3. What are the two things which must be constant for an object to have a constant velocity?

An object must have a constant speed and direction to have a constant velocity.
Exploration
Equipment for this lab includes a small tube, string, an assortment of masses, and a rubber stopper. Tie the rubber stopper to one end of the string. Thread the other end of the string through the tube. Tie the free end of the string to the 100 gram mass. Your gadget should look like the diagram shown below:

Hold the tube with your left hand and grasp the 100 gram mass in your right hand. Gently twirl the tube so that the stopper begins spin in circular motion above your head. The diagram on the right shows a picture of the spinning stopper. Make the motion of the spinning stopper as perfectly horizontal (parallel to the ground) as you can. Now, adjust the speed of the stopper so that you can let go of the mass and it won't move up or down. Practice until you feel comfortable.

4. What is the force pulling the 100 gram mass down? This is the same force that is pulling the stopper inward. Show your equation, answer, and units below (Remember the unit for Newton is kg·m/s²).

\[ W = m \times g \]
\[ W = .100\text{kg} \times 9.81\text{m/s}^2 = .981\text{N} \]

5. If the 100 gram mass is not moving up or down, what is the magnitude of the net force acting on it? (What is the magnitude of the vector sum of the forces?)

The magnitude of the vector sum of the forces is zero.

6. You know that there is a force acting downward on the mass due to gravity. Draw a vector showing how this force acts on the stopper while in circular motion.

7. Draw the force that must exist to counterbalance the force of gravity and keep the stopper moving in a circular path with a constant radius. 
*Hint: think back to Newton’s first law.*

You will now perform three experiments to determine the relationship between the variables involved with circular motion.
**Experiment One: Speed (v) and Inward Acting Force (F_i)**

In this experiment you will keep the spinning radius constant and change the weight of the hanging mass.

G. Find the mass of the rubber stopper and record in Table 1.

H. Adjust the string so that the distance between the top of the tube and the middle of the stopper is 0.75m. Fasten a piece of masking tape to the string just below the bottom of the tube (but not touching). When you are whirling the apparatus, keep the tape just below the bottom of the tube to maintain a constant radius of 0.75m.

I. Again, twirl the tube so that the stopper travels in a horizontal circle. Adjust the speed of the stopper so that the tape stays just below the bottom of the tube. Make sure that the tape is not caught on the tube. Practice this until you feel comfortable keeping constant radius.

J. Now, you will measure the amount of time required for the stopper to make 10 revolutions with a radius of 0.75m. One member of the group will rotate the stopper. Another member will measure the time (using a stopwatch) and count the number of revolutions. Practice before you collect any data.

K. Measure the time to complete 10 revolutions twice.

L. Repeat with 120, 150, 170, and 200 g masses. Record your data in Table 1. Show your work for calculating the circumference.

**Table 1.**

<table>
<thead>
<tr>
<th>Hanging mass (kg)</th>
<th>Weight of mass (N)</th>
<th>Time for 10 revolutions (s)</th>
<th>Average time for 1 revolution</th>
<th>Speed of stopper (ms⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100</td>
<td></td>
<td>Trial 1</td>
<td>Trial 2</td>
<td>Average</td>
</tr>
<tr>
<td>0.120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.170</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use a calculator or computer to graph the inward force (the force or gravity) against the speed of the stopper. Put the speed on the horizontal axis and the force on the vertical axis. Select a power fit trend line for your data and display the equation on the graph. Round the coefficient to the nearest whole number.

8. Write the equation of your line [Y = inward force (Fr) and X = speed (v)]

\[ F = v^2 \]

9. What does this equation tell you about the relationship between speed and the inward acting force?

As you increase the inward acting force you square the speed

**Experiment Two: Speed (v) and Radius (r)**

In this experiment you will find the speed of the circling stopper for different radii. All other variables will be held constant.

C. Begin by marking a radius of 1.50m on the string. Twirl the stopper and record the time that was required to make 10 revolutions while keeping the radius constant.

D. Repeat with radii of 1.25, 1.00, 0.75, 0.50m. Complete Table 2. If the times for a given radius are not close, take a third reading.

**Table 2.**

| Constants: | 0.100kg hanging mass |
| Stopper mass | ______kg |

<table>
<thead>
<tr>
<th>Radius (m)</th>
<th>Time for 10 revolutions (s)</th>
<th>Average time for 1 revolution</th>
<th>Circumference</th>
<th>Speed of stopper (ms(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trial 1 Trial 2 Average</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use a calculator or computer to graph the radius against the speed of the stopper. Put the speed on the horizontal axis and the radius on the vertical axis. Select a power fit trend line for your data and display the equation on the graph. Round the coefficient to the nearest whole number.

10. Write the equation of your line \([Y = \text{radius} \ (r) \text{ and } X = \text{speed} \ (v)]\)

\[r = v^2\]

11. What does this equation tell you about the relationship between speed and radius?

As you increase the radius you square the speed.

**Experiment Three: Speed (v) and the Circling Mass (m)**

Now you will find the speed of the circling stopper for various weights of circling masses. All other variables will remain constant.

C. Attach one stopper to the end of the string and a 400g mass to the other. Set the radius at 0.5m. Twirl the stopper and record the time required to make 10 revolutions.

D. Repeat with 2, 3, 4, and 5 stoppers. Each of the stoppers should be the same size, material, and have the same number of holes. Complete Table 3. If the times for a given number of stoppers are not close, take a third reading.

**Table 3.**

<table>
<thead>
<tr>
<th>Number of stoppers</th>
<th>Circling Mass</th>
<th>Time for 10 revolutions (s)</th>
<th>Average for 1 revolution</th>
<th>Speed of stoppers (ms(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trial 1</td>
<td>Trial 2</td>
<td>Average</td>
</tr>
</tbody>
</table>
Use a calculator or computer to graph the radius against the speed of the stopper. Put the speed on the horizontal axis and the circling mass on the vertical axis. Select a power fit trend line for your data and display the equation on the graph. Round the coefficient to the nearest whole number.

12. Write the equation of your line \([Y = \text{circling mass}(m)\text{ and }X = \text{speed}(v)]\)

\[m = \frac{1}{v^2}\]

13. What does this equation tell you about the relationship between speed and radius?

As you increase the circling mass you decrease the speed by a power of two.

**Putting it all together**

14. You will now try to combine your three equations into one that relates \(r\), \(m\), and \(v\), to \(F\). Your result should show \(F\) by itself on one side of the equation. Discuss this with your group.

\[F = mv^2r^{-1}\]

15. Explain why an object moving in a circular path has a constant speed but is accelerating at the same time.

*It is accelerating because the direction is changing.*